

# Spin content from skyrmions with parameters fit to baryon properties

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**Abstract:-** Earlier work reported on the existence of a term within a generalized skyrmion approach that yields appreciable spin content for the proton. Unfortunately there is no accessible experiment that can fix the coefficient of this term directly; plausible but highly uncertain values for it gave a result for the spin content loosely consistent with the currently measured  $\Delta\Sigma = 0.27 \pm 0.13$ . We here attempt to narrow the range of values for this coefficient by performing global fits to all the parameters of the generalized Skyrme lagrangian while requiring reasonable results for the baryon octet and decuplet masses and octet magnetic moments. This requirement fixes the coefficient loosely, and we find that parameter sets that fit the baryon masses and magnetic moments yield proton spin content near  $\Delta\Sigma \sim 0.15$ .

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## 1. Introduction

When the original measurement [1] of the spin content of the proton yielded a result consistent with zero it was deemed to be a merit of the Skyrme model [2] that it also suggested a vanishing value for this quantity. Already in the early work [3] that showed this it was pointed out that some ways of breaking flavor SU(3) could provide nonzero spin content  $\Delta\Sigma$ , but at least in the approach chosen there  $\Delta\Sigma$  proved to be negative. Thus when more recent measurements [4] gave, for all proton data,  $\Delta\Sigma = 0.27 \pm 0.13$  it became urgent to find sources of positive spin content in the general skyrmion approach if that model were to survive [5-9].

Recently a term in a generalized Skyrme lagrangian has been found [10] that can yield sizable positive proton spin content thus making it straightforward to work with a skyrmion that includes this feature. The term  $\mathcal{L}_{6,1}$  in question [see eq. (1)] contains six derivatives of the skyrmion field. For flavor SU(2) it reduces to a form previously put forth [11,12] as a possible device for stabilizing the skyrmion in place of, or alongside, the usual four-derivative term originally proposed by Skyrme [2]. Thus the presence of this term automatically leaves unchanged the many successful features of the model in SU(2). On the other hand for flavor SU(3) it yields new results including the improved proton spin content. Unfortunately a direct determination of the coefficient  $\epsilon_1$  of this term cannot be made from particle decay rates, the relevant transitions either vanishing identically in this model (viz.,  $\eta' \rightarrow 5\pi$ ), being kinematically forbidden (viz.,  $\eta' \rightarrow \eta + 4\pi$ ), or being hopelessly difficult to measure (e.g.,  $\pi\pi \rightarrow \pi\pi\eta\eta'$ ). Thus ref. [10] contented itself with suggesting plausible values for this new coefficient along with the older and better known coefficients for the other terms in the generalized Skyrme lagrangian. These then led to  $\Delta\Sigma \sim 0.3$  to  $0.4$  which is roughly consistent with current experiment [4].

It is our purpose here to try to narrow the range of acceptable values for  $\epsilon_1$  by carrying out an extensive fit to the masses of the baryon octet and decuplet. The centroids and splittings of the masses for the nonzero strangeness members of these multiplets are sensitive to  $\epsilon_1$  since  $\mathcal{L}_{6,1}$  influences the SU(3) skyrmion mass and moments of inertia [10]. We shall also use the magnetic moments of the octet as a further check on the resulting values for  $\epsilon_1$  as these moments are usually reliably calculated in the skyrmion approach. We do not use here the values for

charge and magnetic radii which are known only for the nucleon.

## 2. Brief review of the formalism

Since the relevant formalism of Yabu and Ando [13] for the flavor SU(3) skyrmion is well known and the modifications required for the inclusion of the six-derivative term with appreciable spin content have already been presented in ref. [10], we sketch only the essential features of the development required to fix notation and set the background of the problem. The generalized skyrmion is described by the lagrangian<sup>1</sup>

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{6,1} + \mathcal{L}_{6,2} + \mathcal{L}_{SB} \\ &= -\frac{F_\pi^2}{16}\text{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\text{tr}[L_\mu, L_\nu]^2 \\ &\quad - \epsilon_1 \frac{g_\omega^2}{m_\omega^2} \text{tr}(B^\mu B_\mu) - \epsilon_2 \frac{g_\omega^2}{2m_\omega^2} \text{tr}(B^\mu) \text{tr}(B_\mu) \\ &\quad + \left[ \frac{F_\pi^2}{32}(m_\pi^2 + m_\eta^2)\text{tr}(U + U^\dagger - 2) + \frac{\sqrt{3}F_\pi^2}{24}(m_\pi^2 - m_K^2)\text{tr}(\lambda_8(U + U^\dagger)) \right], \end{aligned} \quad (1)$$

apart from an anomalous contribution to the  $\eta'$  mass which is not relevant here. We refer to the terms of the lagrangian by their subscripted forms as given in the first line of this equation. Our notation uses the conventional  $L_\mu \equiv U^\dagger \partial_\mu U$  and the somewhat less usual definition

$$B^\mu \equiv -\frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2} L_\alpha L_\beta L_\gamma. \quad (2)$$

In most work on skyrmions a trace is immediately taken over this quantity so that it refers directly to the baryon density, but we need to retain the distinction between terms of the forms of  $\mathcal{L}_{6,1}$  and  $\mathcal{L}_{6,2}$  (i.e., between terms involving one or two traces over the six  $L_\mu$ -operators) which is at the heart of our source for spin content in the skyrmion. Further in eq. (1),  $U(\vec{r}, t)$  is the U(3) chiral field,  $F_\pi$  is the pion decay constant (with experimental value 186 MeV), and  $e$  is the parameter of the four-derivative term introduced by Skyrme in order to stabilize the  $U$ -field.

The first two terms in eq. (1),  $\mathcal{L}_2 + \mathcal{L}_4$ , are the original Skyrme lagrangian [2]. The next terms  $\mathcal{L}_{6,1}$  and  $\mathcal{L}_{6,2}$  involve six derivatives and have been used in the

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<sup>1</sup> We note that a general analysis of effective chiral lagrangians with six-derivative terms has recently become available [14].

past as possible  $\omega$ -coupling repulsive terms for stabilizing the skyrmion [11,12]. They are equivalent [12] in SU(2) but not in SU(3), reminiscent of well-known terms with four derivatives that are equivalent in SU(2) but different in SU(3) [15]. The coefficients of these terms,  $\epsilon_1 g_\omega^2/m_\omega^2$  and  $\epsilon_2 g_\omega^2/2m_\omega^2$ , are taken in a form that allows easy comparison with the  $\omega NN$  coupling constant: we keep  $\epsilon_1 + \epsilon_2 = 1$ , absorbing overall strength into  $g_\omega^2/m_\omega^2$ , with  $m_\omega = 782$  MeV, so that  $\mathcal{L}_{6,1} + \mathcal{L}_{6,2}$  at the SU(2) level continues to give the usual  $\omega$  coupling. The flavor symmetry breaking term  $\mathcal{L}_{SB}$  in eq. (1) is well known [13] to be important for work with the SU(3) skyrmion. Last, we note that in eq. (1) we have not retained two further terms involving four derivatives of  $U$  that were entertained in ref. [10] but then dropped there when they proved to have only small impact on the issue of spin content; this omission also avoids problems of terms of fourth order in field time derivatives which otherwise arise.

The proton spin content is generated from  $\mathcal{L}$  of eq. (1) by introducing the U(3) matrix

$$U = \exp \left[ \frac{2i}{F_\pi} \left( \eta' + \sum_{a=1,8} \lambda_a \phi_a \right) \right], \quad (3)$$

where  $\phi_a$  is the pseudoscalar octet and  $\eta'$  is the ninth pseudoscalar meson. It is well known [5,6] that there is no contribution to the spin content from a U(1) axial current that is a complete four-derivative since this vanishes in producing  $\Delta\Sigma$  as an integral over all space. Here we construct the axial current out of a term in the lagrangian of the form [6]

$$\mathcal{L}' = (2/F_\pi) \partial_\mu \eta' J^\mu \quad (4)$$

generated by varying  $\eta'$ . The static hedgehog makes no contribution to spin content [6] and thus to evaluate  $J^\mu$  we use collective coordinates for the time dependence [16]

$$U(\vec{r}, t) = A(t) U_0(\vec{r}) A^\dagger(t), \quad (5)$$

with the hedgehog embedded in SU(3) as

$$U_0 = \exp[i\vec{\lambda} \cdot \hat{\vec{r}} F(r)], \quad (6)$$

where  $F(r)$  is the profile function. The spin content is then

$$\Delta\Sigma = 2\langle p \uparrow | J^3 | p \uparrow \rangle, \quad (7)$$

and the contribution of  $\mathcal{L}_{6,1}$  to this is [10]

$$\langle p \uparrow | J_{6,1}^3 | p \uparrow \rangle = -\frac{1}{2(3\pi)^3} \frac{g_\omega^2}{m_\omega^2} \frac{\epsilon_1}{\beta^4} I, \quad (8)$$

where

$$I \equiv \int_0^\infty r dr \sin^2 \frac{F}{2} \sin F \left( F'^2 - F' \frac{\sin F}{r} + \frac{\sin^2 F}{r^2} \right). \quad (9)$$

The quantity  $1/\beta^2$  is the moment of inertia for the  $SU_L(3)$  Casimir operator  $C_2(SU_L(3))$ . It appears also in the mass expression [13]

$$M = M_{\text{cl}} + \frac{1}{2} \left( \frac{1}{\alpha^2} - \frac{1}{\beta^2} \right) C_2(SU_R(2)) - \frac{3}{8\beta^2} + \frac{1}{2\beta^2} C_2(SU_L(3)) + \frac{1}{2} \gamma (1 - D^{(8)}(A)), \quad (10)$$

along with  $\alpha^2$ , the moment of inertia for the  $SU_R(2)$  Casimir operator  $C_2(SU_R(2))$ . Expressions for both moments of inertia are shown in ref. [13]. To the forms shown there we have now added the additional pieces yielded [10] by  $\mathcal{L}_{6,1} + \mathcal{L}_{6,2}$ . The other terms in eq. (10) involve the skyrmion mass  $M_{\text{cl}}$  and the  $SU(3)$  symmetry breaking term with coefficient  $\gamma$ . Expressions for these quantities are also given in ref. [13]. Last, forms for the baryon charge radii, axial couplings, and magnetic moments are provided by Kanazawa [17] for the minimal Skyrme lagrangian  $\mathcal{L}_2 + \mathcal{L}_4$ . The introduction of the terms  $\mathcal{L}_{6,1} + \mathcal{L}_{6,2}$  in eq. (1) leads to additional contributions to these observables which are shown in the appendix to the present paper. We note that in the present work the calculation of the observables includes the effects of symmetry breaking [13].

Before presenting numerical results it is worth noting that the integral  $I$  appearing in the spin content, eqs. (8) and (9), is an approximate topological constant in the sense that it can be written as

$$\begin{aligned} I &= \int_0^\infty r dr \sin^2 \frac{F}{2} \sin F \left[ \left( F' + \frac{\sin F}{r} \right)^2 - 3F' \frac{\sin F}{r} \right] \\ &= \frac{3\pi}{4} + \int_0^\infty r dr \sin^2 \frac{F}{2} \sin F \left( F' + \frac{\sin F}{r} \right)^2 \\ &\geq 3\pi/4, \end{aligned} \quad (11)$$

where we have used the profile function boundary values  $F(0) = \pi$  and  $F(\infty) = 0$  and also exploited the fact that  $0 < F(r) \leq \pi$  for actual one-baryon profiles. Since the profile function is near these values for a good part of the range of integration one might expect from Taylor-series expansion that the combination  $F' + \sin F/r$  in eq. (11) will be small on average, and indeed carrying out the integration numerically for a wide variety of functions that satisfy the boundary conditions of  $F(r)$  and, like it, fall monotonically from  $r$  near the origin to infinity suggests that the correction to  $I = 3\pi/4$  is generally less than 10%. The case  $I = 3\pi/4$  is

achieved if and only if  $F' = -\sin F/r$  whose solution  $F(r) = 2 \arctan(c/r)$ , where  $c$  is a constant, has the form suggested by an analysis [18] of the skyrmion in terms of adiabatic invariants for large baryon number. As this form is found [18] to be a reasonable approximation even for  $B = 1$ , it is not surprising that numerically  $I \approx 3\pi/4$  for all the cases considered below. This in turn allows us to approximate the overall spin content by

$$\Delta\Sigma \approx -\frac{1}{(3\pi)^3} \frac{g_\omega^2}{m_\omega^2} \frac{\epsilon_1}{\beta^4} \frac{3\pi}{4}. \quad (12)$$

Since the moment of inertia  $1/\beta^2$  as derived from the lagrangian of eq. (1) is moderately sensitive to the value for  $\epsilon_1$  one should not conclude from this that  $\epsilon_1$  is merely a multiplicative factor in the spin content. As  $1/\beta^2$  is fixed by the baryon mass spectrum our fitting procedure largely reduces to extracting  $1/\beta^2$  and subsequently  $\epsilon_1$  from the spectrum and then determining the spin content from eqs. (7)–(9).

### 3. Results and discussion

In fitting the parameters of eq. (1) to data we keep  $\epsilon_1 + \epsilon_2 = 1$  and take  $g_\omega^2/4\pi = 10$ , as implied by [12]  $\omega NN$  coupling and by [11] the decay  $\omega \rightarrow 3\pi$ . The value  $m_K = 640$  MeV generally used in the symmetry-breaking term of eq. (1) is, as usual [13], larger than the experimental one, most likely because of the omission of other possible symmetry-breaking terms [15]. We now vary  $F_\pi$ ,  $e$ , and  $\epsilon_1$  so as to produce reasonable values for baryon masses. In the previous study [10] it was found that  $F_\pi = 130$  MeV and the rather high value  $e = 20$  led to reasonable  $N$  and  $\Delta$  masses and SU(2) nucleon electroweak properties; we then arbitrarily chose  $\epsilon_1 = -0.7$ , which yields<sup>1</sup>  $\Delta\Sigma = 0.43$  but very high masses for the strange baryons. The results of the current fitting procedure for the baryon octet and decuplet masses—after subtraction of the zero-point mass according to the procedure of Yabu and Ando [13]—are shown in table 1 along with the experimental values. In table 2 we show the results for the magnetic moments of the baryon octet. These fits are generally quite good, certainly of the quality usually found with skyrmions. All but the last set of parameters suffer from a value for the pion decay constant  $F_\pi$  which is less than 50 percent of the experimental  $F_\pi = 186$  MeV. This tendency of the skyrmion to require quite low values for  $F_\pi$  is well

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<sup>1</sup> A somewhat different value was shown in ref. [10] due to a numerical error there.

known [13,16,17]. Case 6 in the tables refers to a fit that holds  $F_\pi$  at its physical value (and uses  $m_K = 540$  MeV); the necessary subtraction for the mass spectrum in this case is correspondingly larger [17], and even then the spectrum is rather less satisfactory than for the cases with  $F_\pi$  smaller than 186 MeV. This case has the further drawback that it gives  $g_A = 1.72$  for the proton, which is considerably worse than the values  $g_A \sim 1.24 \pm 0.05$  obtained for cases 1 through 5. All the cases considered here fall within the rough range for the parameter  $e$  provided [20] by  $\pi\pi$  scattering, namely,  $e \approx 5 \pm 2$ . In tables 1 and 2 we show numbers for the root of the summed squared deviations of the calculated from the measured values for the masses and the magnetic moments, respectively. From these values it emerges that, with respect both to masses and to magnetic moments, the best fits are obtained for cases 1 and 4 in the tables. These two fits are not appreciably different from each other, and are both substantially better than the other fits shown.

Table 3 gives values for spin content for the six parameter sets we have considered, as well as providing the moments of inertia  $1/\alpha^2$  and  $1/\beta^2$ . The two preferred fits of tables 1 and 2, namely, cases 1 and 4, have similar values for  $1/\beta^2$  of around  $555 \pm 20$  MeV, and yield  $\Delta\Sigma = 0.17$  and  $0.14$ , respectively. Inspection of table 3 leads us to conclude that, while we achieve only loose bounds on  $\epsilon_1$ , these suffice to localize  $\Delta\Sigma$  as lying, in this model, between 0.0 and 0.6, with a preference for  $\Delta\Sigma \sim 0.15$ .

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## Appendix

We quote here the additional terms for baryon observables that must be added to the expressions of Kanazawa [17] in order to include the effects of  $\mathcal{L}_{6,1} + \mathcal{L}_{6,2}$ ; the notation here follows that of ref. [17]. To  $\Lambda(x)$  and  $G(x)$  of eq. (21) there must be added

$$\Delta\Lambda(x) = \frac{1}{\pi^4} \frac{g_\omega^2}{m_\omega^2} F_\pi^2 e^4 (\epsilon_1 + \epsilon_2) S^4 \dot{F}^2, \quad (\text{A.1})$$

and

$$\Delta G(x) = \frac{1}{9\pi^4} \frac{g_\omega^2}{m_\omega^2} F_\pi^2 e^4 \epsilon_1 x^2 (1 - C) \frac{S^2}{x^2} \left( 2\dot{F}^2 + \frac{S^2}{x^2} \right). \quad (\text{A.2})$$

Last,  $E$  of eq. (24) must be modified by

$$\Delta E = \frac{1}{\pi^4} \frac{g_\omega^2}{m_\omega^2} F_\pi^2 e^4 (\epsilon_1 + \epsilon_2) \int dx x^2 \left[ \dot{F} \frac{S^2}{x^2} \left( \frac{S^2}{x^2} + 2SC \frac{\dot{F}}{x} \right) \right]. \quad (\text{A.3})$$

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TABLE 1  
Parameters for  $\mathcal{L}$  of eq. (1) and corresponding masses

Case	Parameters			Masses [MeV] (octet/decuplet)				$\sqrt{\sum(M_{\text{th}} - M_{\text{exp}})^2}$
	$e$	$\frac{F_\pi}{F_\pi(\text{exp})}$	$\epsilon_1$	$N_8/\Delta_{10}$	$\Lambda_8/\Sigma_{10}$	$\Sigma_8/\Xi_{10}$	$\Xi_8/\Omega_{10}$	
1	6.0	0.44	-1.00	938	1111	1240	1366	98
				1226	1336	1485	1649	
2	6.0	0.40	-1.50	936	1114	1246	1375	120
				1234	1338	1469	1624	
3	6.0	0.48	-0.35	928	1124	1271	1402	131
				1233	1342	1488	1659	
4	6.6	0.44	-0.71	933	1108	1238	1364	99
				1249	1351	1480	1633	
5	5.4	0.44	-1.26	919	1119	1272	1410	159
				1203	1317	1469	1643	
6	8.0	1.00	-1.45	939	1120	1253	1387	120
				1236	1344	1476	1634	
exp*				939	1116	1193	1318	
				1232	1385	1530	1672	

\*Ref. [19]

TABLE 2  
Parameters for  $\mathcal{L}$  and octet magnetic moments

Case	Magnetic moments [ $\mu_N$ ]								$\sqrt{\sum(\mu_{\text{th}} - \mu_{\text{exp}})^2}$
	p	n	$\Lambda$	$\Sigma^+$	$\Sigma^0$	$\Sigma^-$	$\Xi^0$	$\Xi^-$	
1	2.65	-1.73	-0.83	2.80	0.84	-1.10	-1.75	-0.89	0.73
2	2.80	-1.77	-0.90	2.98	0.93	-1.12	-1.86	-0.99	0.93
3	2.54	-1.72	-0.75	2.64	0.75	-1.13	-1.65	-0.79	0.58
4	2.49	-1.57	-0.77	2.65	0.81	-1.02	-1.62	-0.88	0.69
5	2.87	-1.98	-0.88	3.01	0.88	-1.24	-1.90	-0.89	0.93
6	2.33	-1.76	-0.78	2.42	0.72	-0.97	-1.65	-0.65	0.68
exp*	2.79	-1.91	-0.61	2.46		-1.16	-1.25	-0.65	

\*Ref. [19]

TABLE 3  
The moments of inertia  $1/\alpha^2$  and  $1/\beta^2$ ,  $I$  of eq. (9), and the spin content  $\Delta\Sigma$

Case	$1/\alpha^2$ [MeV]	$1/\beta^2$ [MeV]	$I$	$\Delta\Sigma$
1	130	536	2.47	0.17
2	132	615	2.47	0.34
3	130	452	2.47	0.04
4	145	574	2.47	0.14
5	115	473	2.46	0.17
6	130	810	2.49	0.58
exp*				$0.27 \pm 0.13$

\*Ref. [4]